2.0 IDEAL CYCLES IN ENGINES (AIR STANDARD CYCLES)

2.0 Air Standard Cycles

Internal combustion engines are classified into two groups which are: (a) the rotator and (b) the reciprocating internal combustion engines. A good example of the rotary internal combustion engine is the gas turbine. The reciprocating internal combustion engines which are classified mainly into two groups: (i) Spark Ignition engines and (ii) Compression Ignition engines and these group of engines operating on Otto and Diesel cycles respectively.

In these engines, the products of combustion are expelled to its surroundings and this makes these engines operate on open cycles. For each cycles, fresh charge (a mixture of air and fuel) is introduced. To study the operations and performances of these engines, they are represented with theoretical engines operating on thermodynamic cycles and these theoretical engines are referred to as **air standard engines** in which the working fluid is air. In these engines, heat is added from an external source as opposed to burning fuel and a heat sink is provided as opposed to exhaust, thus returning the air back to its original state.

The following assumptions are made for the air standard cycle:

- The working fluid (air) has a constant mass throughout the entire air cycle and the air is taken to be ideal.
- The air maintains a constant specific heat capacity throughout the cycle.
- The combustion process is replaced by a heat transfer process from an external heat source.
- The cycle is completed by the heat transfer to the surrounding in contrast to the exhaust and the intake processes of an actual engine.
- All the processes are internally reversible.

2.1 GAS POWER CYCLES

2.1.1 Otto Cycle

This cycle which is named after the inventor, Nicolaus Otto, whose engine, operated on this principle in 1876.

The diagram for the ideal air standard Otto cycle is shown below:

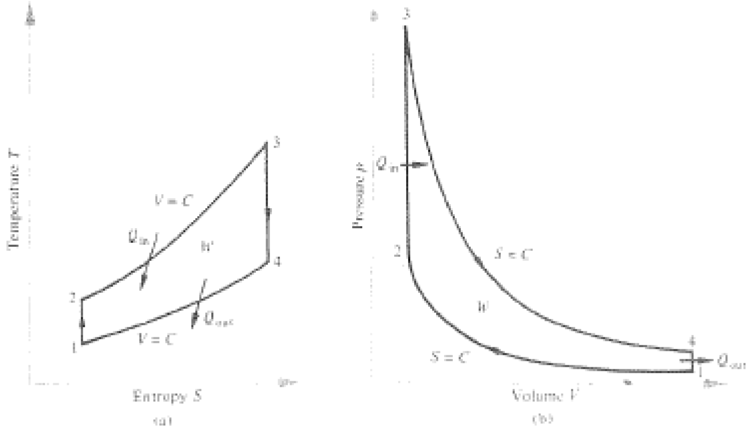


Figure 1: (a) The T-S diagram for an air-standard Otto cycle (b) The P-V diagram for the air-standard Otto cycle

Process 1 -2: Isentropic compression of air takes place from state 1 to state 2.

Process 2-3: constant volume heat addition takes place from state 2 to state3.

Process 3–4: Isentropic expansion occurs from state 3 to state 4.

Process 4-1: heat rejection at constant volume occurs from state 4 to state 1.

Q_{in} is the heat supplied at constant volume

$$Q_{in} = C_v(T_3 - T_2) \tag{1}$$

Heat rejected Q_{out}

$$Q_{out} = C_v(T_4 - T_1) \tag{2}$$

Thermal efficiency $$\eta = 1 - \frac{C_v(T_4 - T_1)}{C_v(T_3 - T_2)} \tag{3}$$

For the isentropic processes, the following expressions hold

$$Tv^{\gamma - 1} = \text{constant} \tag{4a}$$

Where,

γ is the specific heat capacity ratio or adiabatic index of air.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \frac{T_3}{T_4} = \gamma_v^{\ \gamma-1} \tag{4b}$$

Where γ_v = compression ratio

Since,

$$T_3 = T_4 \gamma_v^{\gamma-1} \text{ and } T_2 = T_1 \gamma_v^{\gamma-1} \tag{5}$$

Substituting these expressions into equ. (3), gives:

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_4 - T_1)\gamma_v^{\gamma-1}} \tag{6}$$

Therefore,

$$\eta = 1 - \frac{1}{\gamma_v^{\gamma-1}} \tag{7}$$

QUESTION 1

A petrol engine has a bore 80mm, a stroke of 110mm and a clearance volume of 53.80 cm^3. Calculate thermal efficiency of the petrol engine based on air standard Otto cycle.

Solution

The engine bore D = 80mm = 8 cm = 0.08 m

Engine Stroke L = 110mm = 11cm= 0.11m

Thermal Efficiency $\eta_{th} = 1 - \frac{1}{\gamma_c^{\gamma-1}}$

Total Cylinder Volume $V = V_c + V_s$

The clearance volume V_c

Swept Volume $V_s = \frac{\pi D^2}{4} L$

Compression ratio $\gamma_c = \frac{V}{V_c}$

Where,

γ_c = compression ratio

γ = specific heat ratios

The swept volume $V_s = \frac{\pi (8)^2}{4} 11$= 552.92 cm^3

Total engine volume $V = V_c + V_s$ = 53.8 + 552.92 = 606.72 cm^3

The compression ratio $\gamma_c = \frac{V}{V_c} = \frac{606.72}{53.8}$ = 11.28

γ = 1.4

Thermal Efficiency $\eta_{th} = 1 - \frac{1}{\gamma_c^{\gamma-1}}$ = 1 - $\frac{1}{(11.28)^{1.4-1}}$ = 0.62 or 62%

2.1.2 Diesel Cycle

The following processes take place in an air-standard Diesel cycle:

Process 1 -2: Isentropic compression of air takes place from state 1 to state 2.

Process 2-3: constant pressure heat addition takes place from state 2 to state3.

Process 3–4: Isentropic expansion occurs from state 3 to state 4.

Process 4-1: heat rejection at constant volume occurs from state 4 to state 1.

To calculate the thermal efficiency of the diesel engine, the heat supplied and the heat rejected are required.

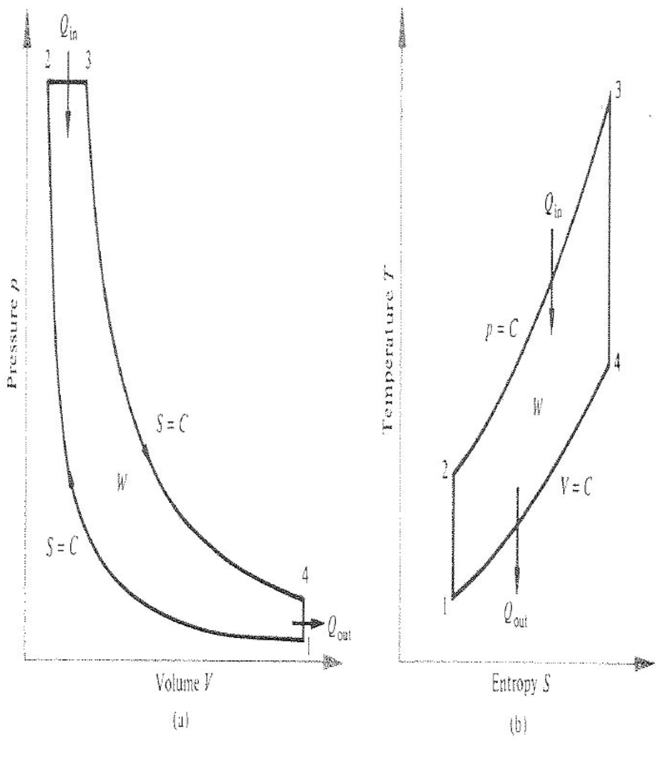


Figure 2: (a) The P-V diagram for the air-standard Diesel cycle (b) The T-S diagram for the air-standard Diesel cycle.

(i) Show that the thermal efficiency (η_{th}) of an engine operating on a diesel cycle is:

$$\eta_{th} = 1 - \frac{1}{\gamma_v^{\gamma-1}}\left(\frac{\gamma_C^{\gamma} - 1}{\gamma(\gamma_C - 1)}\right)$$

Where,

γ_v is the engine compression ratio

γ_C is the cut-off ratio

γ is the ratio of specific heats

$$\eta_{th} = 1 - \frac{Q_2}{Q_1}$$

The heat supplied at constant pressure is given as:

$$Q_1 = mc_p\left(T_3 - T_2\right)$$

The heat rejected at constant volume Q_2 is given as:

$$Q_2 = mc_v\left(T_4 - T_1\right)$$

Substituting (2) and (3) into (1) we have:

$$\eta_{th} = 1 - \frac{c_v\left(T_4 - T_1\right)}{c_p\left(T_3 - T_2\right)} = 1 - \frac{T_1\left(\left(\frac{T_4}{T_1}\right) - 1\right)}{\gamma \cdot T_2\left(\left(\frac{T_3}{T_2}\right) - 1\right)}$$

At the isentropic compression stage,

$$\frac{T_2}{T_1} = \gamma_v^{\ \gamma - 1}$$

Therefore,

$$T_1 = T_2\left(\frac{1}{\gamma_v}\right)^{\gamma - 1}$$

Cut off ratio $\gamma_c = \dfrac{T_3}{T_2}$

$$T_3 = T_2\gamma_c$$

For the isentropic expansion stage:

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = \left(\frac{v_3}{v_2} \cdot \frac{v_2}{v_4}\right)^{\gamma - 1} = \left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma - 1}$$

Since $\frac{v_3}{v_2} = \gamma_c$ and $\frac{v_2}{v_4} = \gamma_v$

Hence $T_4 = T_3\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma-1} = T_2\gamma_c\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma-1}$

Recall,

$$\frac{T_4}{T_1} = \frac{T_2\gamma_c\left(\frac{\gamma_c}{\gamma_v}\right)^{\gamma-1}}{T_2\left(\frac{1}{\gamma_v}\right)^{\gamma-1}} = \gamma_c{}^{\gamma}$$

$$\frac{T_3}{T_2} = \frac{T_2\gamma_c}{T_2} = \gamma_c$$

$$\eta_{th} = 1 - \frac{1}{\gamma_v{}^{\gamma-1}}\left(\frac{\gamma_C{}^{\gamma} - 1}{\gamma(\gamma_C - 1)}\right)$$

QUESTION 2

An engine operates on the air standard diesel cycle. The inlet temperature and pressure are 27˚C and 100KPa respectively. The compression ratio is 12:1 and the heat addition is 1800KJ/kg. Calculate the maximum temperature and pressure of the cycle, the thermal efficiency and the mean effective pressure.

Solution

For the isentropic compression process from state 1 to state 2,

$Pv^{\gamma} = C$ and $Tv^{\gamma-1} = C$

$\gamma = 1.4$

T_1 = 300 K; P_1 = 100kPa; γ_c = 12 and heat supplied Q_{23} = 1800KJ/kg.

To calculate the air temperature at the end of compression,

$T_2 = T_1\gamma_v{}^{\gamma-1} = 300(12)^{1.4-1} = 810.58$ K.

The pressure at the end of the compression stroke P_2 is given as:

$P_2 = P_1\gamma_v{}^{\gamma} = 100(12)^{1.4} = 3242.30 KPa$

The constant pressure heat addition process:

$Q_{23} = c_p(T_3 - T_2) = 1800$ kJ/kg

$$T_3 = T_2 + \frac{1800}{1.005} = 810.58 + 1791.04$$

T_3 = 2601.62 K.

The cycles maximum temperature T_3 = 2601.62 K.

The cycles maximum pressure $P_3 = P_2$ = 3242.30kPa.

The specific volume at the end of injection $v_3 = \frac{RT_3}{P_3} = \frac{(0.287)(2601.62)}{3242.30} = 0.2303 m^3/kg$

For the isentropic expansion process from state 3 to state 4,

$$v_4 = v_1$$

R is the specific gas constant for dry air = 0.287kJkg^{-1}K^{-1}

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(300)}{100} = 0.861 m^3/kg$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1}$$

$$T_4 = 2601.62\left(\frac{0.2303}{0.861}\right)^{1.4-1} = 1535.18K$$

$$P_4 = P_3\left(\frac{v_3}{v_4}\right)^{\gamma} = 3242.30\left(\frac{0.2303}{0.861}\right)^{1.4} = 511.75 \text{ kPa}$$

Heat rejected at constant volume between state 4 and state 1

c_v is the specific heat capacity at constant volume = 0.718kJ/kgK

$$Q_{41} = c_v(T_4 - T_1) = 0.718(1535.18 - 300) = 886.86 kJ/kg$$

The Thermal efficiency of the air standard diesel cycle η_{th}

$$\eta_{th} = \frac{Q_{23} - Q_{41}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{886.86}{1800} = 0.5073 \text{ or } 50.73\%$$

Calculation of the Mean Effective Pressure (MEP)

$$\text{MEP} = \frac{W_{net}}{v_1 - v_2} = \frac{Q_{23} - Q_{41}}{v_1 - v_2}$$

$$v_2 = \frac{v_1}{12} = \frac{0.861}{12} = 0.07175 m^3/kg$$

$$\text{MEP} = \frac{1800 - 886.86}{0.861 - 0.07175} = 1226.93 kPa$$

The Mean Effective Pressure (MEP) = 1226.93kPa.

2.1.3 GAS TURBINE (BRAYTON) CYCLE

The following processes take place in an air-standard Brayton cycle:

Process 1 -2: Isentropic compression of air takes place from state 1 to state 2.

Process 2-3: constant pressure heat addition takes place from state 2 to state 3.

Process 3–4: Isentropic expansion occurs from state 3 to state 4.

Process 4-1: heat rejection at constant pressure occurs from state 4 to state 1.

The Derivation of the Thermal Efficiency of a Brayton Cycle

(a) $\eta_{th} = 1 - \dfrac{Q_2}{Q_1}$

The heat supplied at constant pressure is given as:

$$Q_1 = mc_p\left(T_3 - T_2\right)$$

The heat rejected at constant pressure Q_2 is given as:

$$Q_2 = mc_p\left(T_4 - T_1\right)$$

Substituting (2) and (3) into (1) we have:

$$\eta_{th} = 1 - \frac{c_p\left(T_4 - T_1\right)}{c_p\left(T_3 - T_2\right)} = 1 - \frac{\left(T_4 - T_1\right)}{\left(T_3 - T_2\right)}$$

Pressure ratio $\gamma_p = \dfrac{p_2}{p_1}$

For the isentropic expansion and compression processes,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$T_4 = \frac{T_3}{T_2} \cdot T_1$$

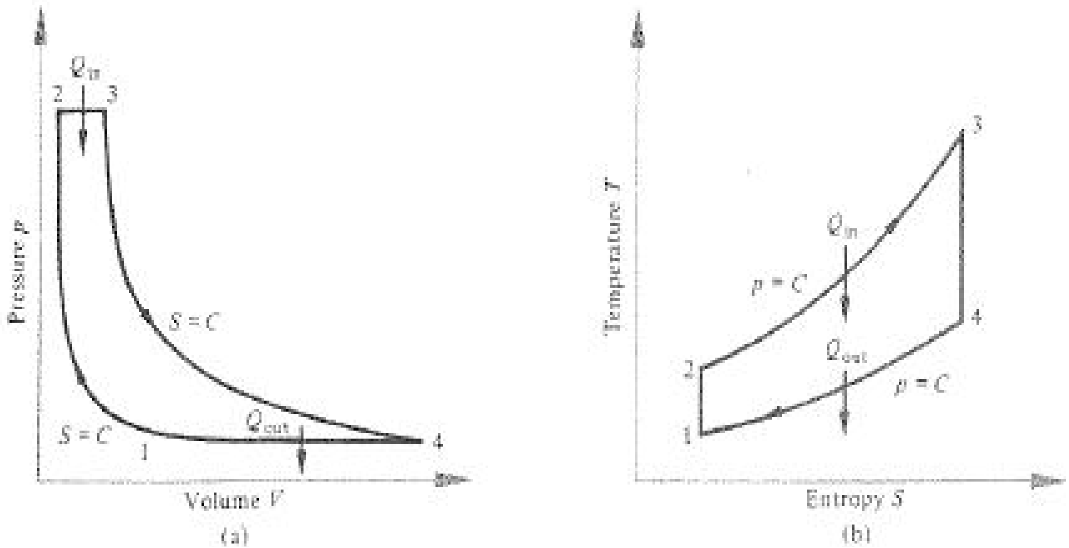
Substituting this into equation (3) we have,

$$\eta_{th} = 1 - \frac{T_1\left(\left(\frac{T_3}{T_2}\right) - 1\right)}{T_2\left(\left(\frac{T_3}{T_2}\right) - 1\right)} = 1 - \frac{T_1}{T_2}$$

$$\gamma_p = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \gamma_p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\eta_{th} = 1 - \frac{1}{\gamma_p^{\left(\frac{\gamma-1}{\gamma}\right)}}$$



QUESTION 3

(1) An air standard Brayton cycle has air entering the compressor at 25˚C and 100KPa. The pressure ratio of the system is 15 and the maximum allowable temperature in the cycle is 1500˚C.

Calculate;

(a) The pressure and temperature of each state in the cycle.

(b) The compressor work, turbine work, the thermal efficiency and the work ratio.

Solution

PV diagram

P_1 = 100kPa, T_1 = 25 ˚C =298 K, $\gamma_p = \frac{p_2}{p_1} = 15$

$$T_2 = T_1 \gamma_p^{\left(\frac{\gamma-1}{\gamma}\right)} = 298(15)^{\left(\frac{0.4}{1.4}\right)} = 646\,\text{K}$$

$p_3 = p_2 = 1500$ kPa, $T_3 = 1500$ ˚C = 1773 K

$$T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{\left(\frac{\gamma - 1}{1.4} \right)} = 1773 \left(\frac{1}{15} \right)^{\left(\frac{0.4}{1.4} \right)} = 817.9 \text{ K}$$

$P_4 = p_1 = 100$ kPa

(b) From the first law,

Compressor work $W_{comp} = c_p (T_2 – T_1) = 1.005(646 – 298) = 349.74$ kJ/kg

Turbine work $W_{tur} = c_p (T_3 – T_4) = 1.005(1773 – 817.9) = 959.9$ kJ/kg

Net work $W_{net} = W_{tur} - W_{comp} = 610.14$ kJ/kg

Heat supplied $Q_1 = h_3 – h_2 = c_p (T_3 – T_2) = 1.005 (1773 – 646) = 1132.64$ kJ/kg

Thermal Efficiency $\eta_{th} = \frac{W_{net}}{Q_1} = \frac{610.14}{1132.64} = 0.539$

Work Ratio = $\eta_{th} = \frac{Net\_Work}{Gross\_Work} = \frac{W_{net}}{W_{tur}} = \frac{610.14}{959.9} = 0.636$